**Blind man and pills**

**Problem:**

A blind man is alone on a deserted island. In his pocket he has two blue pills and two red pills. He must take exactly one red pill and one blue pill today and one red pill and one blue pill tomorrow or else he will die. He cannot distinguish which pill is red and which pill is blue. How can he solve this problem?

**Solution:**

Firstly, he breaks a pill in half, and as he does this, he pops one half in his mouth and saves the other half for tomorrow. When he has done this with all four pills, he will have consumed one red pill and one blue pill and he will have the same leftover (exactly one red and one blue pill). He just consumes the rest tomorrow and he is saved.

**Barber paradox**

**Problem:**

The paradox goes like this: There is a barber who shaves only those who don’t shave themselves (barber who shaves all those, and those only, who do not shave themselves). The question is, does the barber shave himself or not?

Any answer to this question results in a contradiction. If the barber shaves himself he cannot shave himself because he shaves only those who don’t shave themselves. If he doesn’t shave himself he must shave himself. It seems like there is no right answer.

**Solution:**

So, is there a solution? Yes, there is. The problem is that we assume there exists such barber. If we look at the definition of the barber (insert logical definition in the slides) we can immediately see a problem with the bi-implication when person “***y”*** is the barber. We want both the statement and its negation to be true which cannot happen by (definition of negation). So, the barber does not exist (and cannot exist) but we’re asking a question about him.

**Fork in the road**

**Problem:**

You reach a fork in the road. A sign explains that in one direction is Heaven and the other is Hell. Each path is blocked by a Guard. The sign goes on to say that the guards know which path leads to where one of the guards will always lie and the other will always tell the truth, but it does not say which guard is which.

You may ask only one question to only one guard in order to determine, with certainty, the way to Heaven. What is that question?

There is another variation of this problem which I like more. Imagine that you have to use the toilet really badly. And one road leads to a toilet and the other doesn’t. What question should you ask a guard so that you don’t shit yourself 😊.

**Solution:**

The question which gives you the answer to which path is which is:

***“If I ask the other guard “Is your path, the path to the toilet?” what would his answer be?”***

If they answer **False/No**, then we know the path to the toilet is at other guard. If they answer **True/Yes**, we know the path to the toilet is at the guard we just asked.

Note: We don’t actually get to know which guard is which. We only get information for the two paths. (Insert graphic with all possibilities)

**Navigating an Infinitely Dense Minefield**

This one I especially like.

**Problem:**

Consider a unit square **[0, 1] × [0, 1]**. I place a landmine at every point with rational coordinates (so at ***(a, b)*** for ***a, b ∈ Q***. Can you find a continuous path in the square from ***(0, 0)*** to ***(1, 1)*** that does not pass through any mines? (a continuous path is one that you can draw without lifting your pen from the paper).

**Solution:**

At first this seems impossible because every point in the square either contains a mine or is infinitely close to a mine.

However, you suddenly remember the lectures of associate professor Minko Markov. He taught us that there exists a bijection between the rational numbers and the natural numbers (in other words there are the same quantity of natural numbers and rational numbers; they are both countable infinities). But ordered pairs of naturals are also countable and it follows that the ordered pairs of rational numbers are also countable. But then you also remember that the real numbers greater than 0 are uncountable, so at last the ordered pairs of rational numbers are less than the real numbers greater than 0.

In short there are only countably infinite mines on the minefield.

Then we ask the question: **How many lines pass through the origin and through the unit square?** The answer, of course, is infinitely many but the more important thing is that this is an uncountably infinity (Why: take all linear functions that pass through the origin). That means there are (actually uncountably infinitely many lines) that don’t pass through any point with rational coordinates. We do the same trick for a line going through the point ***(1, 1)*** and we have a solution.

So an example solution will be: